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IFπGP Closed Sets in Intuitionistic Fuzzy Topological Spaces

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Abstract— *In this paper we introduce a new class of intuitionistic fuzzy set called intuitionistic fuzzy π generalized pre closed sets and π generalized pre open sets in intuitionistic fuzzy topological spaces. After giving the fundamental definitions we have discussed the various properties and examples. Also we have discussed some applications of π generalized pre closed sets in intuitionistic fuzzy topological spaces.*

Keywords— *Intuitionistic fuzzy topology, π generalized pre closed sets, π generalized pre open sets in intuitionistic fuzzy, intuitionistic fuzzy $\pi p T_{1/2}$ space and intuitionistic fuzzy $\pi gp T_{1/2}$ space.*

I. Introduction

The concept of fuzzy sets was introduced by Zadeh [10] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Sarsak and Rajesh [7] introduced π generalized semi pre closed sets.

In this paper we introduce intuitionistic fuzzy π generalized semi closed sets and intuitionistic fuzzy π generalized semi open sets and study some of their properties.

II. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. We denote the set of all intuitionistic fuzzy sets in X, by IFS (X).

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined as follows:

(i) $int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

(ii) $cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5:[6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$

(ii) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$

(iii) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$

(iv) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq int(cl(A))$

(v) intuitionistic fuzzy α closed set (IFαCS in short) if $cl(int(cl(A))) \subseteq A$

(vi) intuitionistic fuzzy α open set (IFαOS in short) if $A \subseteq int(cl(int(A)))$.

Definition 2.6:[7] Let $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS of an IFTS (X, τ). Then the semi closure of A (scl(A) in short) is defined as $scl(A) = \cap \{ K / K$ is an IFSCS in X and $A \subseteq K \}$.

Definition 2.7:[7] Let A be an IFS of an IFTS (X, τ). Then the semi interior of A (sint(A) in short) is defined as $sint(A) = \cup \{ K / K$ is an IFSOS in X and $K \subseteq A \}$.

Definition 2.8:[9] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$.

(ii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$.

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Definition 2.9:[9] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if cl((A) ⊆ U whenever A ⊆ U and U is an IFOS in X.

Definition 2.10:[7] Let A be an IFS of an IFTS (X, τ). Then the alpha closure of A (αcl(A) in short) is defined as αcl(A) = ∩ { K / K is an IFαCS in X and A ⊆ K }.

Definition 2.11:[7] Let A be an IFS of an IFTS (X, τ). Then the alpha interior of A (αint(A) in short) is defined as αint(A) = ∪ { K / K is an IFαOS in X and K ⊆ A }.

Definition 2.12:[7] Let A be an IFS in (X, τ) , then

(i) α cl(A) = A ∪ cl(int(cl(A)))
(ii) αint(A) = A ∩ int(cl(int(A)))

Definition 2.13:[7] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy alpha generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X.

Definition 2.14:[2] Let A be an IFS of an IFTS (X, τ). The pre interior of A (pint(A) in short) is defined by the union of all fuzzy pre-open sets of X which are contained in A. The intersection of all fuzzy pre-closed sets containing A is called the pre-closure of A and is denoted by (pcl(A) in short)

pint(A) = ∪ { G / G is an IFPOS in X and G ⊆ A }

pcl(A) = ∩ { K / K is an IFPCS in X and A ⊆ K }.

Result 2.15: [8] If A is an IFS in X, then pcl(A) = A ∪ cl(int(A)).

III. Intuitionistic Fuzzy Π Generalized Pre Closed Set

Definition 3.1: An IFS A is said to be an intuitionistic fuzzy π-generalized pre-closed set (IFπGPCS in short) in (X,τ) if pcl(A) ⊆ U whenever A ⊆ U and U is an IFπOS in X. The family of all IFπGPCSs of an IFTS (X,τ) is denoted by IFπGPC(X).

Example 3.2: Let X = $\{a,b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.3, 0.1), (0.7, 0.6) ⟩. Then the IFS A = ⟨ x, (0.2, 0.1), (0.8, 0.7) ⟩ is an IFπGPCS in X.

Theorem 3.3:

(i) Every IFCS is an IFπGPCS but not conversely.
(ii) Every IFαCS is an IFπGPCS but not conversely.
(iii) Every IFGCS is an IFπGPCS but not conversely.
(iv) Every IFRCS is an IFπGPCS but not conversely.
(v) Every IFPCS is an IFπGPCS but not conversely.
(vi) Every IFαGCS is an IFπGPCS but not conversely

Proof (i): Let A be an IFCS in X and let A ⊆ U and U is an IFπOS in (X,τ). Since pcl(A) ⊆ cl(A) and A is an IFCS in X, pcl(A) ⊆ cl(A) = A ⊆ U. Therefore A is an IFπGPCS in X.

Proof (ii): Let A be an IFαCS in X and let A ⊆ U and U is IFπOS in (X,τ). By hypothesis, cl(int(cl(A))) ⊆ A. Since A ⊆ cl(A), cl(int (A)) ⊆ cl(int(cl(A) ⊆ A. Hence pcl(A) ⊆ A ⊆ U. Therefore, A is an IFπGPCS in X.

Proof (iii): Let A be an IFGCS in X and let A ⊆ U and U is an IFπOS in (X,τ). Since pcl(A) ⊆ cl(A) and by hypothesis, pcl(A) ⊆ U. Therefore, A is an IFπGPCS in X.

Proof(iv): Let A be an IFRCS in X. By definition 2.9, A = cl(int(A)). This implies that cl(A) = cl(int(A)). Therefore cl(A) = A. That is A is an IFCS in X. By theorem 3.3, A is an IFπGPCS in X.

Proof(v): Let A be an IFPCS in X and let A ⊆ U and U is an IFπOS in (X,τ). By definition 2.5, cl(int(A) ⊆ A. This implies that pcl(A) = A ∪ cl(int(A) ⊆ A. Therefore pcl(A) ⊆ U. Hence, A is an IFπGPCS in X.

Proof(vi): Let A be an IFαGCS in X and let A ⊆ U and U is an IFπOS in (X,τ). By definition 2.14,
A ∪ cl(int(cl(A))) ⊆ U. This implies that cl(int(cl(A))) ⊆ U and cl(int(A) ⊆ U. Therefore pcl(A) = A ∪ cl(int((A)) ⊆ U. Hence, A is an IFπGPCS in X.

Example(i): Let X = $\{a,b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.8, 0.7) ⟩. Then the IFS A = ⟨ x, (0.2, 0.2), (0.8, 0.7) ⟩ is an IFπGPCS in X but not an IFCS in X.

Example(ii): Let X = $\{a,b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.1, 0.2), (0.5, 0.6) ⟩. Then the IFS A = ⟨ x, (0.2, 0.3), (0.6, 0.5) ⟩. is an IFπGPCS in X but not an IFαCS in X, since cl(int(cl(A) = ⟨ x, (0.5, 0.6), (0.1, 0.2) ⟩ $\not\subseteq$ A.

Example(iii): Let X = $\{a,b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.4, 0.6) ⟩. Then the IFS A = ⟨ x, (0.1, 0.2), (0.7, 0.8) ⟩. is an IFπGPCS but not an IFGCS in X, since A ⊆ T but cl(A) = ⟨ x, (0.4, 0.6), (0.2, 0.3) ⟩ $\not\subseteq$ T.

Example(iv): Let X = $\{a,b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.1, 0.1), (0.8, 0.9) ⟩. Then the IFS A = ⟨ x, (0.1, 0.2), (0.7, 0.8) ⟩ is an IFπGPCS but not an IFRCS in X, since cl(int(A) = ⟨ x, (0.8, 0.9), (0.1, 0.1) ⟩ ≠ **A.**

Example(v): Let X = $\{a,b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.6, 0.7) ⟩. Then the IFS A = ⟨ x, (0.3, 0.4), (0.5, 0.6) ⟩ is an IFπGPCS but not an IFPCS in X, since cl(int(A) = ⟨ x, (0.6, 0.7), (0.2, 0.3) ⟩ $\not\subseteq$ **A.**

Example(vi): Let X = $\{a,b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.5, 0.6), (0.5, 0.4) ⟩. Then the IFS A = ⟨ x, (04, 0.5), (0.6, 0.5) ⟩ is an IFπGPCS but not an IFαGCS in X, since αcl(A) = $1_\sim$ $\not\subset$ **T.**

Proposition 3.4: IFSCS and IFπGPCS are independent to each other.

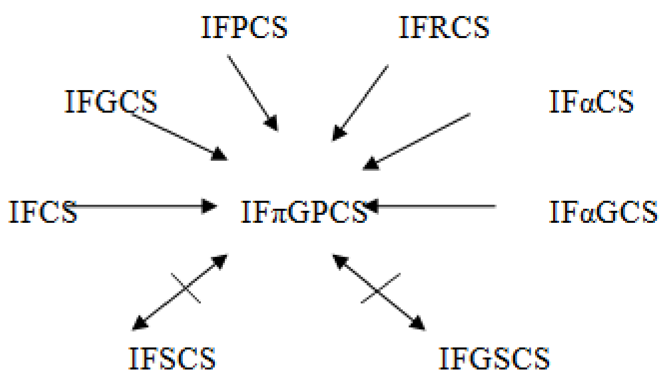
Example 3.5: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T_1, T_2, T_3, T_4, 1_\sim\}$ be an IFT on X, where T_1 = ⟨ x, (0.1, 0.3), (0.4, 0.3) ⟩, T_2 = ⟨ x, (0, 0.2), (0.2, 0.3) ⟩, T_3 = ⟨ x, (0, 0.2), (0.3, 0.3) ⟩, T_4 = ⟨ x, (0.1, 0.3), (0.2, 0.3) ⟩. Then the IFS A = ⟨ x, (0.1, 0.3), (0.2, 0.3) ⟩ is an IFSCS but not an IFπGPCS in X, since A ⊆ T but pcl(A) = ⟨ x, (0.2, 0.3), (0.1, 0.3) ⟩ $\not\subseteq$ T.

Example 3.6: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T_1, T_2, T_3, T_4, 1_\sim\}$ be an IFT on X, where T_1 = ⟨ x, (0, 0.2), (0.1, 0.2 ⟩, T_2 = ⟨ x, (0.1, 0.4), (0.4, 0.3) ⟩, T_3 = ⟨ x, (0.2, 0.4), (0.4, 0.5) ⟩, T_4 = ⟨ x, (0, 0.1), (0.5, 0.5) ⟩. Then the IFS A = ⟨ x, (0.1, 0.2), (0.4, 0.5) ⟩ is an IFGSCS but not an IFπGPCS in X, since A ⊆ T but pcl(A) = ⟨ x, (0.1, 0.2), (0.2, 0.4) ⟩ $\not\subseteq$ T.

Proposition 3.7: IFGSCS and IFπGPCS are independent to each other.

Example 3.8: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.4), (0.6, 0.5) ⟩. Then the IFS A = ⟨ x, (0.2, 0.3), (0.6, 0.5) ⟩ is an IFπGPCS but not an IFGSCS in X since scl(A) $\not\subseteq$ T.

The following implications are true.



In this diagram A ⟶ B means that A implies B but not conversely and A ⟷ B means A and B are independent of each other. None of them is reversible.

Remark 3.9: The union of any two IFπGPCS is not an IFπGPCS.

Example 3.10: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.4, 0.5) ⟩. Then the IFSs A = ⟨ x, (0, 0.1), (0.4, 0.6) ⟩, B = ⟨ x, (0.3, 0.3), (0.4, 0.5) ⟩ are IFπGPCSs but A ∪ B is not IFπGPCS in X.

IV. Intuitionistic Fuzzy Π-Generalized Pre-Open Sets

In this section we introduce intuitionistic fuzzy π generalized pre open sets and studied some of its properties.

Definition 4.1: An IFS A is said to be an intuitionistic fuzzy π-generalized pre-open set (IFπGPOS in short) in (X, τ) if the complement A^c is an IFπGPCS in X. The family of all IFπGPOSs of an IFTS (X, τ) is denoted by IFπGPO(X).

Example 4.2: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.3, 0.1), (0.7, 0.6) ⟩. Then the IFS A = ⟨ x, (0.8, 0.7), (0.2, 0.1) ⟩ is an IFπGPOS in X.

Theorem 4.2: For any IFTS (X, τ), we have the following:

(i) Every IFOS is an IFπGPOS.
(ii) Every IFSOS is an IFπGPOS.
(iii) Every IFαOS is an IFπGPOS.
(iv) Every IFPOS is an IFπGPOS.

But the converses are not true.

Proof: Straight forward. The converse of the above statements need not be true, which can be seen by the following examples.

Example 4.3: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.8, 0.7) ⟩. Then the IFS A = ⟨ x, (0.8, 0.7), (0.2, 0.2) ⟩ is an IFπGPOS in X but not an IFOS in X.

Example 4.4: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T_1, T_2, T_3, 1_\sim\}$ be an IFT on X, where T_1 = ⟨ x, (0.1, 0.3), (0.3, 0.5) ⟩, T_2 = ⟨ x, (0.1, 0.2), (0.4, 0.5) ⟩, T_3 = ⟨ x, (0.3, 0.4), (0.3, 0.5) ⟩. Then the IFS A = ⟨ x, (0.4, 0.5), (0.1, 0.2) ⟩ is an IFπGPoS but not an IFSOS in X.

Example 4.5: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.1, 0.2), (0.5, 0.6) ⟩. Then the IFS A = ⟨ x, (0.6, 0.5), (0.2, 0.3) ⟩. is an IFπGPOS in X but not an IFαOS in X.

Example 4.6: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.6, 0.7) ⟩. Then the IFS A = ⟨ x, (0.5, 0.6), (0.3, 0.4) ⟩ is an IFπGPOS but not an IFPOS in X.

Theorem 4.8: Let (X, τ) be an IFTS. If A ∈ IFπGPOS then V ⊆ int(cl(A) whenever V ⊆ A and V is IFCS in X.

Proof: Let A ∈ IFπGPOS. Then A^c is an IFπGPCS in X. Therefore pcl(A^c) ⊆ U whenever A^c ⊆ U and U is IFOS in X. That is cl(int(A^c)) ⊆ U. This implies U^c ⊆ int(cl(A)) whenever U^c ⊆ A and U^c is IFCS in X. Replace U^c = V, we get V ⊆ int(cl(A)) whenever V ⊆ A and V is IFCS in X.

Theorem 4.9: Let (X, τ) be an IFTS. Then for every A ∈ IFπGPO(X) and for every B ∈ IFS(X) , pint(A) ⊆ B ⊆ A implies B ∈ IFπGPO(X).

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Proof: By hypothesis, $A^c \subseteq B^c \subseteq (pint(A))^c$. Let $B^c \subseteq U$ and U be an IFOS. Since $A^c \subseteq B^c$, $A^c \subseteq U$. But A^c is an IFπGPCS, $pcl(A^c) \subseteq U$. Also $B^c \subseteq (pint(A))^c = pcl(A^c)$ (By Theorem). Therefore, $pcl(B^c) \subseteq pcl(A^c) \subseteq U$. Hence B^c is an IFπGPCS, which implies B is an IFπGPOS of X.

Remark 4.10: The intersection of any two IFπGPOS is not an IFπGPOS in general.

Example 4.11: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.4, 0.5) ⟩. Then the IFSs A = ⟨ x, (0.4, 0.6), (0, 0.1) ⟩ , B = ⟨ x, (0.4, 0.5), (0.3, 0.3) ⟩ are IFπGPOSs but A ∩ B is not IFπGPOS in X.

Theorem 4.12: An IFS A of an IFTS (X, τ) is an IFπGPOS iff $F \subseteq pint(A)$ whenever F is an IFCS and $F \subseteq A$.

Proof: Necessity: Suppose A is an IFπGPOS in X. Let F be an IFCS and $F \subseteq A$. Then F^c is an IFOS in X such that $A^c \subseteq F^c$. Since A^c is an IFπGPCS, we have $pcl(A^c) \subseteq F^c$. Hence $(pint(A))^c \subseteq F^c$. Therefore $F \subseteq pint(A)$.

Sufficiency: Let A be an IFS of X and let $F \subseteq pint(A)$ whenever F is an IFCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is an IFOS. By hypothesis, $(pint(A))^c \subseteq F^c$. Which implies $pcl(A^c) \subseteq F^c$. Therefore A^c is an IFπGPCS of X. Hence A is an IFπGPOS of X.

Corollary 4.13: An IFS A of an IFTS (X, τ) is an IFπGPOS iff $F \subseteq int(cl(A))$ whenever F is an IFCS and $F \subseteq A$.

Proof: Necessity: Suppose A is an IFπGPOS in X. Let F be an IFCS and $F \subseteq A$. Then F^c is an IFOS in X such that $A^c \subseteq F^c$. Since A^c is an IFπGPCS , we have $pcl(A^c) \subseteq F^c$. Therefore $cl(int(A^c)) \subseteq F^c$. Hence $(int(cl(A)))^c \subseteq F^c$. This implies $F \subseteq int(cl(A))$.

Sufficiency: Let A be an IFS of X and let $F \subseteq int(cl(A))$ whenever F is an IFCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is an IFOS. By hypothesis, $(int(cl(A)))^c \subseteq F^c$. Hence $cl(int(A^c)) \subseteq F^c$, which implies $pcl(A^c) \subseteq F^c$.Hence A is an IFπGPOS of X.

Theorem 4.14: For an IFS A, A is an IFOS and an IFπGPCS in X iff A is an IFROS in X.

Proof: Necessity: Let A be IFOS and an IFπGPCS in X. Then $pcl(A) \subseteq A$. This implies $cl(int(A)) \subseteq A$. Since A is an IFOS, it is an IFPOS. Hence $A \subseteq int(cl(A))$. Therefore A = int(cl(A)). Hence A is an IFROS in X.

Sufficiency: Let A be an IFROS in X. Therefore A = int(cl(A)). Let $A \subseteq U$ and U is an IFOS on X. Thisimplies $pcl(A) \subseteq A$. Hence A is an IFπGPCS in X.

V. Applications Of Intuitionistic Fuzzy Π Generalized Pre Closed Sets

Definition 5.1: An IFTS (X, τ) is said to be an intuitionistic fuzzy ${}_{\pi p}T_{1/2}$ (IF ${}_{\pi p}T_{1/2}$ in short) space if every IFπGPCS in X is an IFCS in X.

Definition 5.2: An IFTS (X, τ) is said to be an intuitionistic fuzzy ${}_{\pi gp}T_{1/2}$ ($IF_{\pi gp}T_{1/2}$ in short) space if every IFπGPCS in X is an IFPCS in X.

Theorem 5.3: Every $IF_{\pi p}T_{1/2}$ space is an $IF_{\pi gp}T_{1/2}$ but the converse is not true.

Proof: Let X be an $IF_{\pi p}T_{1/2}$ space and A be an IFπGPCS in X. By hypothesis, A is an IFCS in X. Since, every IFCS is an IFPCS, A is an IFPCS in X. Hence X is an $IF_{\pi gp}T_{1/2}$ space.

Example 5.4: Let X = $\{a, b\}$ and let τ = $\{0_\sim, T, 1_\sim\}$ be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.8, 0.7) ⟩. Then, (X, τ) is an $IF_{\pi gp}T_{1/2}$ space. But it is not an $IF_{\pi p}T_{1/2}$ space, since the IFS A = ⟨ x, (0.2, 0.2), (0.8, 0.7) ⟩ is an IFπGPCS but not an IFCS in X.

Theorem 5.5: Let (X, τ) be an IFTS and X is an $IF_{\pi p}T_{1/2}$ space, then

(a) Any union of IFπGPCS is an IFπGPCS.
(b) Any intersection of IFπGPOS is an IFπGPOS.

Proof: (a) Let $\{A_i\}_{i\in J}$ is a collection of IFπGPCSs in an $IF_{\pi p}T_{1/2}$ space (X, τ). Therefore, every IFπGPCS is an IFCS. But the union of IFS is an IFCS. Hence the union of IFπGPCS is an IFπGPCS in X. (b) It can be proved by taking complement in (a).

Theorem 5.6: An IFTS X is an $IF_{\pi gp}T_{1/2}$ space iff IFGPO(X) = IFPO(X).

Proof: Necessity: Let A be an IFπGPOS inX, then A^c is an IFπGPCSs in X. By hypothesis A^c is an IFPCS in X. Therefore, A is an IFPOS in X. Hence IFGPO(X) = IFPO(X).

Sufficiency: Let A be an IFπGPCS in X. Then A^c is an IFπGPOSs in X. By hypothesis A^c is an IFPOS in X. Therefore, A is an IFPCS in X. Hence X is an $IF_{\pi gp}T_{1/2}$ space.

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