

Edge Detection Algorithms Using Brain Tumor Detection and Segmentation Using Artificial Neural Network Techniques

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Abstract— Brain tumor is one of the major causes of death among people. It is evident that the chances of survival can be increased if the tumor is detected and classified correctly at its early stage. The segmentation of brain tumors in magnetic resonance images (MRI) is a challenging and difficult task because of the variety of their possible shapes, locations, image intensities. In this research, it is intended to summarize and compare the methods of automatic detection of brain tumor through Magnetic Resonance Image (MRI) using Histogram Thresholding and Artificial Neural Network. The proposed method can be successfully applied to detect the contour of the tumor and its geometrical dimension. Also in this research, a modified Artificial Neural Network (ANN) model that is based on learning vector quantization with image and data analysis and manipulation techniques is proposed to carry out an automated brain tumor classification using MRI-scans. The assessment of the modified ANN classifier performance is measured in terms of the training performance, classification accuracies and computational time. MRI (Magnetic resonance Imaging) brain tumor images detection is a difficult task due to the variance and complexity of tumors. This research presents two techniques for the detection purpose; first one is Histogram Thresholding and second is Artificial Neural Network technique. The Edge detection segmentation of brain tissue in the magnetic resonance image (MRI) is very important for detecting and existence of outlines the brain tumor. In this research an algorithm for segmentation based on the symmetry character of brain image is presented. Our goal is to detect the position and edge of tumors automatically. Experiments were carried on real pictures, and the results show that the algorithm is flexible and convenient. This proposed method is more efficient and faster to identify the detecting the tumor region from T1, T2-weighted MRI brain images. The proposed Neural Network technique consists of some stages, namely, feature extraction, dimensionality reduction, detection, segmentation and classification. In this paper, the proposed method is more accurate and effective for the brain tumor detection and segmentation. For the implementation of this proposed work we use the Image Processing Toolbox under Matlab Software.

Keywords— Artificial neural network (ANN), edge detection, image segmentation, brain tumor detection, histogram thresholding.

I. INTRODUCTION

Brain has a very complex structure and is considered as a kernel part from the body. Nature has tightly safeguarded the brain inside a skull that hinders the study of its function as well as makes the diagnosis of its diseases more intricate. But, brain is not prone to diseases and can be affected by the abnormal growth of the cells in that change its normal structure and behavior; a disease generally known as a brain tumor. Brain tumors either include tumors in the central spinal canal or inside the cranium. Automatic defects detection in MRI is quite useful in several diagnostic and therapeutic

applications computed tomography and MRI are two imaging modalities that help researchers and medical practitioners to study the brain by looking at it non-invasively. Most of the time, the tumor segmentation and classification become harder due to quantity of MR images and blurred boundaries. Since brain is safeguarded by the skull, therefore, an early detection of brain tumor is only possible when diagnostic tools are directed at intracranial cavity. MRI is a medical imaging technique, and radiologists use it for visualization of the internal structure of the body. MRI can provide plentiful of information about human soft tissues anatomy as well as helps diagnosis of brain tumor. MR images are used to analyze and study behavior of the brain. A powerful magnetic field is used to align the nuclear magnetization of hydrogen atoms (or protons) of water in the body. In the presence of RF (radio frequency) electromagnetic fields, hydrogen nuclei produce a rotating magnetic field which is detectable by the scanner. Since protons can absorb energy at specific frequency and have the ability to reemit that energy; therefore, a transmitter coil is normally fitted around the human skull to measure the net magnetization. The transmitter coil functions in the following way: first, it produces electromagnetic waves and transmits these waves inside the brain, and then a receiver coil measures the intensity of the emitted electromagnetic waves. Digital Image Processing (DIP) refers to processing of digitized images through computers and finds varieties of applications in the most diverse areas of business, science and technology. Any typical image processing application comprises of extraction of important features of a given image, using which later description, interpretation, or understanding of an image is provided. DIP techniques are used to solving varieties of problems related to image processing, office and industrial automation, remote sensing, science, criminology, image transmission and storage, astronomy, space, meteorology, information technology, entertainment, consumer electronics, printing, graphic arts and defense. Digital images are captured through imaging devices and cover almost the entire frequency spectrum, ranging from gamma to radio waves. Moreover, an additional gradient coil is used for spatial localization of the signal. Lastly, the recorded signals (or electromagnetic waves) are reconstructed into an image by a specialized computer program. Early detection and classification of brain tumors is very important in clinical practice. Many researchers have proposed different techniques for the classification and detection of brain tumors based on different sources of information. In this paper we propose a process for brain tumor classification and detection,

focusing on the analysis of Magnetic Resonance (MR) images. Our aim is to achieve a high accuracy in discriminating the two types of tumors through a combination of several techniques for image segmentation, feature extraction and classification.

The proposed technique has the potential of assisting clinical diagnosis. Necessary preprocessing steps prior to characterization and analysis of regions of interest (ROIs) are segmentation and registration. Image registration is used to determine whether two subjects have ROIs in the same location. However, in this work we do not take into account the location of the tumor in the classification model so we do not employ registration. Image segmentation is required to delineate the boundaries of the ROIs ensuring, in our case, that tumors are outlined and labeled consistently across subjects. Segmentation can be performed manually, automatically, or semi-automatically. The manual method is time consuming and its accuracy highly depends on the domain knowledge of the operator. Specifically, various approaches have been proposed to deal with the task of segmenting brain in MR images. The performance of these approaches usually depends on the accuracy of the spatial probabilistic information collected by domain experts. Utilizing the statistical information cumulated during the segmentation process, this method can provide satisfying results even in cases where the boundaries of the ROIs cannot be easily identified.

II. REVIEW OF RELATED WORKS

The literature survey carried out related to technology impact in the study of brain related diseases revealed that a fair amount of research has gone into this area. Analysis and diagnosis of various brain related diseases like brain stroke using neural network [1], atherosclerotic disease in human carotid arteries [2], basal ganglia for accurate detection of human spongiform encephalopathy [3], brain tumors [4], Alzheimer's disease (AD) [5], brain infarct, infection, hamartoma, and tumor [6], neurosarcooidosis (NS) [7], cystic or necrotic brain tumors [8], different pathologic situations [9], nodular enhancement of the oculomotor nerve [10], Hemangioblastoma (Tumors) of the conus medullaris [11], MS lesions [12], [13], Parkinson's disease [14], pathological/normal brain [15], HIV/AIDS [16] are being cited in literature on processing of Brain MRI images. Brain MRI segmentation (for different applications) by applying different techniques such as nonparametric density estimation [17], Topology-preserving, anatomy-driven segmentation (TOADS) [18], atlas-based whole brain segmentation method with an intensity renormalization procedure [19], a knowledge-driven algorithm [20], tractography techniques [21], fuzzy logic [22-24], self-organizing map (SOM) neural network [25], k-means objective function combined genetic algorithm [26], Hidden Markov Model (HMM) [27], analysis of brain MRI data using registration based on deformation tensor morphometry [28], learning-based method [29], active markers [30] are being cited in the literature.

We have come across works like detection of brain activation using conditional random field (CRF) [31], age-related changes brain white matter (WM) [32], analyzing

regions of neuronal activation [33], brain development and fetal brain pathology [34], effect of caffeine on verbal working memory task [35], neural correlates of retrieval success for music memory [36] and early functional brain development with data collected from children during natural sleep [37]. Extraction of texture properties of the brain's white matter (WM) [38], spherical wavelet transformation to extract shape features of cortical surfaces [39], single cell detection [40], Bayesian decision theory applied to brain tissue classification [41], computation and visualization of volumetric white matter connectivity in diffusion tensor (DT) MRI [42], topological visualization of human brain diffusion MRI [43], labeling of structures in 3D brain MRI data sets using expert anatomical knowledge that is coded in fuzzy sets and fuzzy rules [44] are found in literature.

III. PROPOSED TECHNIQUE

This section illustrates the overall technique of our proposed Brain Tumor Detection and Segmentation Using Histogram Thresholding and Artificial Neural Network Techniques. In this research, the proposed system is a modified version of the Artificial Neural Network. The modification is based on automatic utilization of specified regions of interest (ROIs) within the tumor area in the MRI images using Histogram Thresholding technique. Form each ROI, set of extracted features include tumor shape and intensity characteristics are extracted and normalized. Each ROI is then given a weight to estimate the PDF of each brain tumor in the MR image. These weights are used as a modeling process to modify the Artificial Neural Network. The presented work is based upon Histogram Thresholding and Artificial Neural Network for brain image segmentation and brain tumor detection. The image of the brain is acquired through MRI technique. If the histograms of the images corresponding to the two halves of the brain are plotted, symmetry between the two histograms should be observed due to symmetrical nature of the brain along its central axis. On the other hand, if any asymmetry is observed, the presence of the tumor is detected. After detection of the presence of the tumor, Thresholding can be done for segmentation of the image. Segmentation is done by calculating the threshold point. The differences of the two histograms are plotted and the peak of the difference is chosen as the threshold point. Using this threshold point, the whole image is converted into binary image providing with the boundary of the tumor. The binary image is now cropped along the contour of the tumor to calculate the physical dimension of the tumor. The whole of the work has been implemented using MATLAB 2012. There are four major steps in the proposed approach for brain tumor classification:

- (a) Edge detection and ROI segmentation: delineating the boundary of the tumor (ROI) in an MR image.
- (b) Feature extraction: getting meaningful features of the ROI identified in the previous step.
- (c) Feature selection: removing the redundant features.
- (d) Classification: learning a classification model using the features.

The proposed algorithm starts by reading the input image, converting it to grey scale image then applying image segmentation techniques for extracting the Region of Interest (ROI). A set of reference MRIs is taken as the training database. Feature vectors are extracted for each image in the training set during the training phase. In the testing phase, the feature vector of the test image is computed. The proposed approach is evaluated on real images, and the results are compared with other algorithms, in particular ANN algorithm. During the segmentation process, each image region confined by a rectangular window is represented by a feature color.

The main objectives of proposed work are given below:

1. Canny's edge detection algorithm is computationally more expensive compared to Sobel, Prewitt and Robert's operator. However, the Canny's edge detection algorithm performs better than all these operators under almost all scenarios.
2. Histogram Thresholding based image segmentation is the process of partitioning a digital image into multiple segments (sets of pixels, also known as super pixels). The goal of segmentation is to simplify and change the representation of an image into something that is more meaningful and easier to analyze.
3. Histogram Thresholding-based methods are very efficient when compared to other image segmentation methods because they typically require only one pass through the pixels. In this technique, a histogram is computed from all of the pixels in the image, and the peaks and valleys in the histogram are used to locate the clusters in the image.
4. Image size is the big problem in the Matlab. In our proposed approach we first considered that the MRI scan images are either color, Gray-scale or intensity images herein are displayed with a default size. In proposed work the MR image are always taken as default size, so this is free from the size of the database images.
5. The objective of image segmentation is to cluster pixels into prominent image region. In this research, segmentation of Gray level images is used to provide information such as anatomical structure and identifying the

Region of Interest i.e. locate tumor, lesion and other abnormalities. The proposed approach is based on the information of anatomical structure of the healthy parts and compares it with the infected parts.

6. There are different types of noise encountered by different techniques, depending on the noise nature and characteristics, namely Gaussian noise and impulse noise. In this paper we assumed that the main image noise is additive and random.
7. In proposed work, other than filtering the region of interest (ROI) is proposed to identify different tumor types and/or different infected areas. It also introduced to enhance the processing time by executing the features processing algorithm in the identified areas instead of the whole image frame. Three are some parameters given below:

A. Magnetic Resonance Imaging

Magnetic Resonance Imaging (MRI) is an imaging technique used primarily in medical settings to produce high quality images of the inside of the human body. In order to preview about MRI, in this section we give a brief description

of the principles of MRI which are referred. In MRI, the image is a map of the local transverse magnetization of the hydrogen nuclei.

This transverse magnetization in turn depends on several intrinsic properties of the tissue. The MR phenomenon relies on the fundamental property that protons and neutrons that make up a nucleus possess an intrinsic angular momentum called spin. When protons and neutrons combine to form nucleus, they combine with oppositely oriented spins. Thus, nuclei with an even number of protons and neutrons have no net spin, whereas nuclei with an odd number of protons or neutrons possess a net spin. Hydrogen nuclei have an NMR signal since its nucleus is made up of only a single proton and possess a net spin. The human body is primarily fat and water, which have many hydrogen atoms. Medical MRI primarily images the MRI signal from the hydrogen nuclei in the body tissues. The net spin of the nucleus around its axis gives it an angular moment. Since the proton is a positive charge, a current loop perpendicular to the rotation axis is also created, and as a result the proton generates a magnetic field. The joint effect of the angular moment and the self generated magnetic field gives the proton a magnetic dipole moment parallel to the rotation axis. Under normal condition, one will not experience any net magnetic field from the volume since the magnetic dipole moments are oriented randomly and on average equalize one another.

B. Histogram Thresholding

In image processing, the Histogram Thresholding method (HT) is a very simple method used for automatic image Thresholding. Like 'Otsu's Method' and the 'Iterative Selection Thresholding Method', this is a histogram based Thresholding method. This approach assumes that the image is divided in two main classes: The background and the foreground. The HT method tries to find the optimum threshold level that divides the histogram in two classes.

C. Artificial Neural Network (ANN)

An Artificial Neural Network (ANN) is an information processing paradigm that is inspired by the way biological nervous systems, such as the brain, process information. The key element of this paradigm is the novel structure of the information processing system. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. ANNs, like people, learn by example. An ANN is configured for a specific application, such as pattern recognition or data classification, through a learning process.

Learning in biological systems involves adjustments to the synaptic connections that exist between the neurons. This is true of ANNs as well.

IV. EVALUATION AND RESULTS

To verify the effectiveness (qualities and robustness) of the proposed tumor detection, we conduct several experiments with this procedure on several images. In this work we load an MRI image and apply the different technique on loaded image

in the Image Processing Toolbox under the Mat lab Software. Below steps of our proposed work is given:

Step 1: Firstly we develop a particular GUI for this implementation. After that we develop a code for the loading the MR image in the Mat lab database.

Step 2: Develop a code for the edge detection process. For the edge detection process we use the canny edge detection technique. After that we apply the edge detection on the loaded MR image and we got the edge image.

Step 3: Develop a code for the segmentation using histogram thresholding. Image segmentation is the process of

partitioning a digital image into multiple segments (sets of pixels, also known as super pixels). The goal of segmentation is to simplify and/or change the representation of an image into something that is more meaningful and easier to analyze. So we apply the segmentation on the MR image.

Step 4: After that we develop code for Detection using ANN we can easily do the classification of the MR image and detect the tumor present in image. The main figure window of our proposed method is given below:

Automated brain tumor images segmented and edge Detection identity.

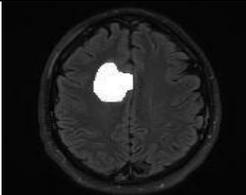
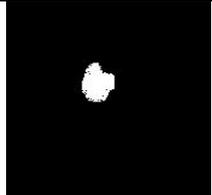
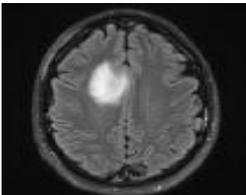
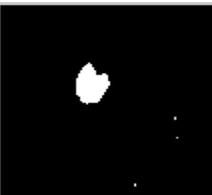
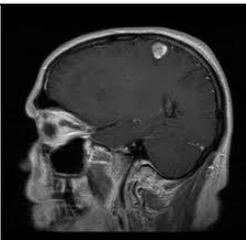
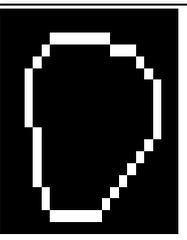
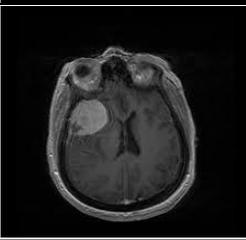
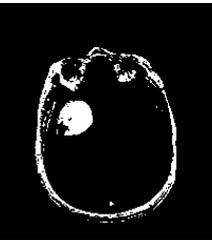
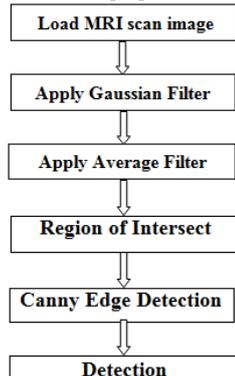
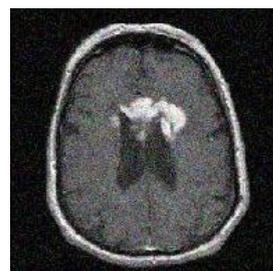
Input Image	Tumor Detection	Boundry Extraction	Size (m*n)
			Name aa Size 64x56 Class uint8 Bytes 3584 N
			Name bb Size 116x87 Class uint8 Bytes 10092
			Name cc Size 21x22 Class uint8 Bytes 462
			Name dd Size 47x44 Class uint8 Bytes 2068

Fig. 1. Main figure window.

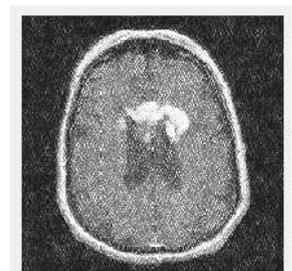
Flow chart of proposed method



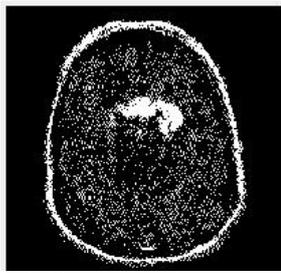
V. RESULTS



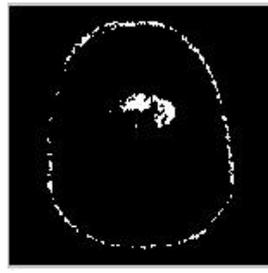
1. MRI scan of brain.



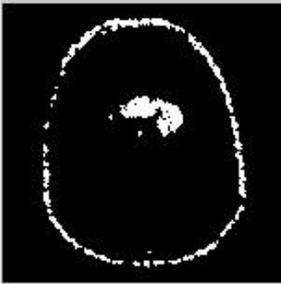
2. Enhanced image fig.



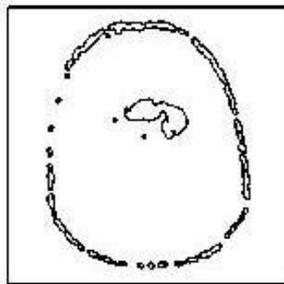
3. Binary to gray image.



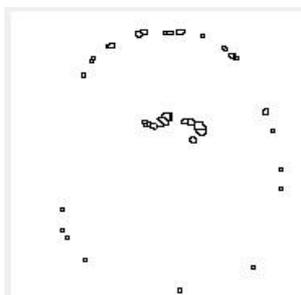
4. Result after erosion applied.



5. Result after dilation.



6. Final output.



7. Detection.

VI. CONCLUSION AND FUTURE SCOPE

The research proposes a method for classification of tumor in a brain image. The main objective of this step is to differentiate the different abnormal brain images based on the optimal feature set. This classification is performed on proton Magnetic Resonance Spectroscopy images. But the classification accuracy results are different for different datasets which is one of the drawbacks of this approach. Experiments are conducted on various real world datasets and the results concluded that the proposed algorithm yield good results when compared with the other classifiers. The results revealed that the proposed hybrid approach is accurate, fast and robust. In this research, we proposed two approaches for Brain tumor detection, identification and classification. The first approach is based on an integrated set of image processing algorithms, while the other is based on a modified and improved probabilistic artificial neural networks structure. The proposed integrated image processing algorithm is based on a modified canny edge detection algorithm and implemented using MATLAB. However, simulation results using this algorithm showed its ability to accurately detect and identify the contour of the tumor, its computational time and accuracy were much less than its corresponding algorithms that use the parallel distributed processing nature of neural

networks to reduce computing time and enhance the classification accuracy. This led us to propose a modified and improved probabilistic artificial neural networks structure. The modification is based on automatic utilization of specified regions of interest (ROIs) within the tumor area in the MRI images. From each ROI, set of extracted features include tumor shape and intensity characteristics are extracted and normalized. Each ROI is then given a weight to estimate the PDF of each brain tumor in the MR image. These weights are used as a modeling process to modify the conventional PNN.

The work on image analysis is typically highly interdisciplinary. It will draw on results from multivariate statistics, numerical analysis, linear algebra, wavelet theory, MR physics, computational geometry, computer science, computer graphics, etc. In this future work, the technologies like matlab tool, Artificial Neural Network (ANN), Fuzzy Logic, Genetic, edge detection and k-means Algorithm comparison, Hidden Markov Model (HMM), Support Vector Machine (SVM), Wavelets, Level sets will be explored to interpret, analyze, and recognize the various brain diseases. the proposed work aims at recognition, interpretation, and analysis of MRI brain tumor images, segmentation concerned with diseases such as tuberculoma, neurocysticercosis, infarction, hematoma, tumor and multiple sclerosis. It aims to provide fast, automatic methods and to introduce more robustness and regularity for interpretation of these diseases.

The scope of future work we proposed different types of approaches Future research in the segmentation of the brain tumor images will strive towards improving the accuracy, precision and computational speed of segmentation methods, as well as reducing the amount of manual interaction. Computational efficiency will be particularly important in the real time processing applications.

Possibly the most important question surrounding the use of image segmentation is its application in clinical settings. Computerized segmentation methods have already demonstrated their utility in research applications and are now increasingly in use for computer aided diagnosis and the radiotherapy planning. It is unlikely that automated image segmentation methods will ever replace physicians but they will likely become crucial elements of the tumor image analysis. Segmentation methods will be particularly valuable in areas such as computer integrated surgery, where visualization of anatomy is a critical component.

The scope of future work includes, as proposed method uses single slice of T2 and contrast enhanced T1. It can be tested on other slices. The scope of future project encompasses segmenting the whole tumor by using a single technique of MRI. It can be done through in depth study of texture and intensities variations. The main objective is to design such a system that could be able to segment out all kind of brain tumor with accuracy and fast processing time.

This work can be extended to the 3D visualization of tumor, with its volume representation. It can be conducted with the softwares 3D Slicer and the SPM. The later is supported by Matlab.

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